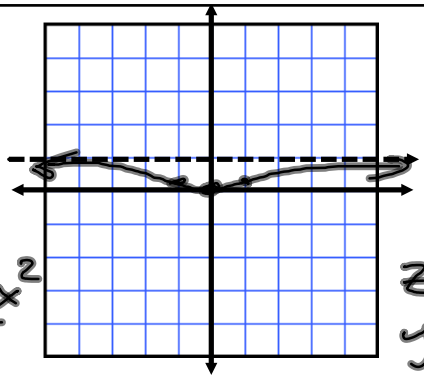
7) $y = \frac{x^2}{x^2+3}$



domain: $\mathbb{R}$

asymp:
h → y = 1
v → none

zeros: {0}

y-int: (0,0)

max: none

min: (0,0)

c.p.

$\frac{dy}{dx} = \frac{2x(x^2+3) - 2x \cdot x^2}{(x^2+3)^2}$

$0 = \frac{6x}{(x^2+3)^2}$ c.p. 0

1st Test
$f'(-1) = -$
$f'(0) = 0$
$f'(1) = +$

increasing $(0, \infty)$

decreasing $(-\infty, 0)$

concave up $(-1, 1)$

concave down $(-\infty, -1) \cup (1, \infty)$

inflection
$(-1, \frac{1}{4})$
$(1, \frac{1}{4})$

points of inflection

$f'(x) = \frac{6x}{(x^2+3)^2}$

$f''(x) = \frac{6(x^2+3)^2 - 6x(2(x^2+3) \cdot 2x)}{(x^2+3)^4}$

$= \frac{6x^2+18-24x^2}{(x^2+3)^3} = \frac{-18x^2+18}{(x^2+3)^3} = \frac{-18(x^2-1)}{(x^2+3)^3}$

$0 = (x^2-1) = (x+1)(x-1)$

± 1

11) $g(x) = x - \frac{8}{x^2} = \frac{x^3-8}{x^2}$

$x - 8x^{-2}$

domain: $\mathbb{R} \setminus \{0\}$

range

c.p.

$g'(x) = 1 + \frac{16}{x^3} = \frac{x^3+16}{x^3}$

$0 = x^3 + 16$

$-16 = x^3$

$\sqrt[3]{-16} = x$

asymp:

v → $x = 0$

h → none

zeros: $\{2\}$

y-int: none

max: $(-2.519, -3.782)$

min: none

increasing $(-\infty, -2.519) \cup$

decreasing $(0, \infty)$

$(-2.519, 0)$

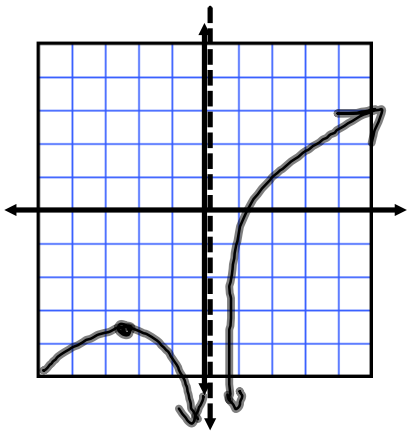
2nd derivative $g''(x) = \frac{-48}{x^4}$

test $g''(\sqrt[3]{-16}) = \frac{-48}{(\sqrt[3]{-16})^4} < 0 \rightarrow$ max

Points of inflection

$0 = \frac{-48}{x^4}$ undef at $x = 0$

$g''(1) = -48$



15) $y = \dfrac{x^2-6x+12}{x-4}$

domain: $\mathbb{R}$ ex $\{4\}$

range:

$$\frac{dy}{dx} = \frac{(2x-6)(x-4)-(x^2-6x+12)}{(x-4)^2}$$

$$= \frac{2x^2-14x+24-x^2+6x-12}{(x-4)^2}$$

$$= \frac{x^2-8x+12}{(x-4)^2}$$

C.P.

$0 = x^2-8x+12$

$0 = (x-6)(x-2)$

$x = 2, 6$

asymp:

h → none

v → $x = 4$

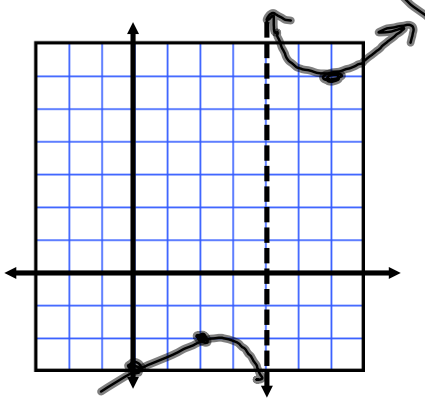
zeros: none

y-int: $(0,-3)$

max: $(2,-2)$

min: $(6,6)$

inflection: $(-2, -\frac{28}{6})$



$F'(1) = +$

$F'(3) = -$

$$\frac{d^2y}{dx^2} = \frac{(2x-8)(x-4)^{\cancel{x}} - 2\cancel{(x-4)}(x^2-8x+12)}{(x-4)^{\cancel{x}3}}$$

$$= \frac{2x^2-20x+32-2x^2+16x-24}{(x-4)^3} = \frac{-4x+8}{(x-4)^3}$$

$$F''(2) = \frac{-4(2)+8}{(2-4)^3} =$$

$$F''(6) = \frac{-4(6)+8}{(6-4)^3} = +$$

$F''(2) = 0$

$-4x-8=0$

$-4x=8$

$x=2$